



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

BYERLY'S FOURIER'S SERIES AND SPHERICAL, CYLINDRICAL, AND ELLIPSOIDAL HARMONICS.*

The workers in the fields of mathematical science may be roughly classified under three heads. First, there are the pure mathematicians, who cultivate the science with little or no regard to its applications. Many of these devotees, in fact, are willing to pursue lines of investigation which do not obviously lead anywhere; and some few possess or affect a contemptuous disregard for utility in research. They revel in the intricacies of pure analysis and transcendental geometry. Not content with the space and space relations presented to us in nature, they explore all sorts of hypothetical regions and deduce results which would be interesting and possibly useful if humanity were otherwise constituted than it is and existed elsewhere than it does. They work only for the enlargement and refinement of the mathematical mill; whether their attachments will enhance its general efficiency is a question of no importance to them.

Secondly, there is another extreme class, including many physicists, chemists, geologists, etc., who are busied with the investigation of phenomena which must ultimately be reduced to mathematical expression, but which are not yet wholly brought within the domain of competent theories. The devotees of this class are profoundly impressed with the facts they observe. But they distrust mathematical processes and frequently plume themselves on their freedom from all restraints of theory. They cannot listen to the "music of the spheres," especially the music of the molecular spheres of modern physics, in the presence of mathematical machinery. They abominate the details of precise calculations, and are generally content to express the results of their observations by the graphical process, which is not infrequently worked by them to an ingeniously profitless extent. They are quick to discover the salient features and relations of phenomena, but their work usually falls somewhat short of exact generalizations.

Thirdly, between these two extremes there is a smaller class now commonly called mathematical physicists. Their object, like that of the second class, is the interpretation of natural phenomena. But they work always by the aid of mathematical theories, and are content only when they discover and correlate quantitative relations under such theories. Their interest in pure mathematics is restricted to what is obviously useful; and they look with little satisfaction on those branches of the science which have not passed the fact-

* An Elementary Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics, with applications to Problems in Mathematical Physics. By William Elwood Byerly, Ph. D., Boston, U. S. A.: Ginn & Company, Publishers, 1893.

naming and curve-tracing stage. They are the utilitarians and organizers, and to their labors, chiefly, are due the permanent advances in applied mathematics.

Such being the diversity of aims and tastes amongst the workers in mathematical science, it is not often that a new book is of special interest to more than one of the classes named. The subject of this notice, however, is an exception, and must be a source of interest and delight to students in whatever field they may work. Fourier's series, spherical, cylindrical, and ellipsoidal harmonics involve an array of exquisite analysis for the pure mathematician; they afford expression to a wide variety of physical phenomena in ways that cannot fail to be interesting and instructive to the experimentalist; and they indicate the roads along which the mathematical physicist may expect to make further advances.

There is a growing demand for such a book as Prof. Byerly has produced. Hitherto, most students have approached the subject through the original papers of Fourier, Poisson, and Laplace, or through the more recent memoirs of Dirichlet and Riemann. The great work of Heine and the highly condensed chapters in Thomson and Tait's *Natural Philosophy* are too difficult, and the later works of Todhunter and Ferrers are too special and juiceless for the beginner.

In his plan of presenting the subjects Prof. Byerly has done well, we think, to follow the example of Fourier and Riemann. To the expert this plan may sometimes appear prolix, but is admirably adapted to awaken interest and fix the ideas. It leads directly and naturally from the characteristic differential equations and the conditions of problems to the final integrals. The wonderful properties of these integrals make their discovery one of the most delightful and profitable of studies when the road by which they are reached is not too obscure and circuitous.

The book consists of nine chapters and an appendix. The first of these gives an excellent introduction to the subjects considered, a sort of bird's-eye view, from which the student may get at the outset a fairly good idea of the nature and drift of the enquiry. Chapters II and III are devoted to questions which lead up to Fourier's series and to the proofs of its generality and validity. From a didactic point of view these are the most important chapters in the book; and the author has chosen wisely probably in following pretty closely Dirichlet and Riemann, since the methods of these authorities meet the approval of the majority of mathematicians. We may venture the opinion, however, that Poisson's method of dealing with this delicate branch of analysis is still worthy of study. Indeed, it would seem that what DeMorgan said more than fifty years ago does not now need modification—"Further to verify

the preceding methods," (of development of functions in trigonometric series) "I add one which is of frequent use in the writings of Poisson, and which I consider much the best adapted of any to give a sound view of the subject, as soon as the new and difficult considerations which it introduces have become familiar."*

Chapter IV is devoted to applications, wherein numerous capital illustrations of the use of Fourier's series and integrals are worked out. Chapters V and VI treat of zonal and spherical harmonics; VII of cylindrical harmonics (Bessel's functions); and VIII of curvilinear co-ordinates and ellipsoidal harmonics. The final chapter, by Dr. Maxime Bôcher, gives an interesting though brief history of progress in the development of the subject. The appendix gives useful numerical tables of surface zonal harmonics, hyperbolic functions, roots of Bessel's functions, and values of the latter functions themselves.

A noteworthy feature of the work, a feature which helps much to render it an elementary work in the best sense, is the number and variety of applications carefully explained in full. This, added to the author's direct and clear demonstrations, make the book an uncommonly readable and useful one. He who cannot catch the spirit of the harmonic analysis by the aid of such a book is a hopeless case.

We have a few small faults to record, in the hope that their mention here may be of use to future book-makers. The first of these relates to the matter of notation. The tendency seems to be inevitable in favor of using the symbol ∂ to indicate partial differentiation; and whatever may be the intrinsic merits of the notation used by the author, it would have been better, we think, for the cause of science, "to fall in with the procession," and write, for example,

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

instead of

$$D_t u = a^2 D_x^2 u.$$

Secondly, although many numerical examples and their answers are given in the book, we have not discovered the numerical details of any one of them. This is a defect common to most treatises, except those devoted to astronomy and geodesy, wherein the art of computing is indispensable. The wayfaring student, whose arithmetic is generally the weakest element in his equipment, is always greatly aided in his journey to a new mathematical region by the elaborate details of a few numerical examples.

Lastly, so good and useful a book would have been better and more useful if it had been supplemented by an index.

R. S. WOODWARD.

* DeMorgan, Differential and Integral Calculus, p. 614.